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On the concept of the magnetic viscosity: analytical expression for the time dependent magnetization

D.V. Berkov [1]

Physics Dept, Keele University, Keele, UK

Exact analytical expression for the time-dependent magnetization of the system with known energy barrier distribution is obtained. The relation between the obtained result and usual definition of the magnetic viscosity is discussed. Corrections to the 'critical volume approach' are shown to be of order $\sigma_{\Delta E}/T$, where $\sigma_{\Delta E}$ is the width of the energy barrier distribution density.

Time-dependent magnetic phenomena have been intensively studied recent years [1–5] for their importance in various pplications of magnetic systems. The assumption, that the (practically) irreversible changes of the magnetization take place due to the equilibrium thermodynamical fluctuations in the system, leads to the expression

$$M(t) = M_0\ e^{-\gamma(E)t} \tag{1}$$

with the single relaxation speed $\gamma(E) = \gamma_0 \exp(-E/T)$ for the system with the single barrier energy E. Real systems always have considerable energy barrier distribution and do not obey the simple exponential law (1).

For the system with the normalized energy barrier distribution density $\rho(E)$ the general expression for the reduced magnetization $m(t) = (M(t) + M_s)/(M_0 + M_s)$ (M_0 denotes the magnetization for $t = 0$, M_s is the saturation magnetization) is

$$m(t) = \int_{E_{min}}^{E_{max}} \rho(E)\ e^{-\gamma(E)t}\ dE. \tag{2}$$

Correspondence to: D.V. Berkov, Physics Department, Keele University, Keele, Staffordshire ST5 5BG, UK.

[1] Permanent address: Institute of Chem. Phys., 142432 Chernogolovka, Russia.

The first practically useful expression was obtained from the general equation (2) by Street and Wooley [6] in 1949 and is used up to now by most experimentalists because of its surprisingly good agreement with the experimental results for many different systems:

$$m(t) = C - S\ \ln(t), \tag{3}$$

where S is called the magnetic viscosity.

Strictly speaking, eq. (3) was derived by Street and Wooley for a "very special distribution" $\rho(E)$ [7]. Namely, the analysis of assumptions made in ref. [6] shows that (3) is valid for (i) the energy independent density $\rho(E)$, i.e., $\rho(E) = \text{const} > 0$ for $E_{min} < E < E_{max}$ and $\rho(E) = 0$ otherwise and (ii) for the time region $\tau_0 \exp(E_{min}/T) = t_{min} \ll t \ll t_{max} = \tau_0 \exp(E_{max}/T)$, where $\rho_0 = 1/\gamma_0$.

The expression (3) (and hence – the concept of the magnetic viscosity by itself) was criticized by Aharoni [7], because it "obviously breaks down (diverges) for large and small t and cannot be transformed back to (1) for the limit of the narrow distribution $\rho(E)$". Both reproaches are incorrect: the equation obtained for the restricted time region $t_{min} \ll t \ll t_{max}$ (see above) obviously cannot account for very small and very large times and cannot be reduced to the case of narrow energy distribution $E_{min} \to E_{max}$, because in

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this case there is no time region, where (3) is valid.

Aharoni's suggestion to use the so called gamma-distribution for the relaxation time density, because "then the expression (2) can be integrated analytically", is not very useful too, because it is well known, that for the ill-posed problem like (2) good fits for $m(t)$ can be obtained using almost any kind of $\rho(E)$ distribution, but the width of the distribution which gives the best fit can be very far from the actual distribution width. It can be seen, for example, from the results published in ref. [2], where for the given set of experimental data good fits were obtained both for gamma- and uniform distributions of relaxation times with very different widths.

Nevertheless, the questions, why does eq. (3) work so well in many experimental situations and what is the nature of corrections (if there are any) to the linear logarithmic behavior are still under study. In this paper we derive the exact analytical expression for the time dependent magnetization in terms of the energy barrier distribution density and discuss the connection between our result and the usual definition of the magnetic viscosity (3).

We start with the general equation (3), which can be rewritten in the form

$$-\frac{\mathrm{d}m(\tau)}{\mathrm{d}\tau}=\int_{-\infty}^{+\infty}\rho(E)\Gamma(E)\,\mathrm{e}^{-\Gamma(E)\tau}\,\mathrm{d}E, \qquad (4)$$

where we have introduced the dimensionless time $\tau = t\gamma_0$ and relaxation speed $\Gamma(E)=\exp(-E/T)$ and have expanded the integral limits for the simplicity of analytical calculations, using the fact, that $\rho(E)=0$ for $E<E_{\min}$ and $E>E_{\max}$.

Due to the exponentially rapid variation of the relaxation speed $\Gamma(E)$ with E the kernel of the integral (4)

$$K(E,\tau)=\Gamma(E)\,\mathrm{e}^{-\Gamma(E)\tau}$$

has a sharp peak with the width $\sim T$ at the point $E_c = \Gamma \ln \tau$, where $K(E,\tau)$ has its maximum value $K_{\max} = 1/\mathrm{e}\tau$. The Taylor expansion of $\rho(E)$ near $E = E_c$ converts (4) into

$$-\frac{\mathrm{d}m(\tau)}{\mathrm{d}\tau}=\frac{T\rho(E_c)}{\tau}+\sum_{k=1}^{\infty}\frac{\rho^{(k)}(E_c)}{k!}\times\int_{-\infty}^{+\infty}(E-E_c)^k\Gamma(E)\,\mathrm{e}^{-\Gamma(E)\tau}\,\mathrm{d}E \qquad (5)$$

from which usually only the first term of the right side is taken into account. This term corresponds to the called "critical volume approach" [4,5], which assumes that for the given time . moments of all particles with the energy barrier $E<E_c(t)$ have already made the transition in the field direction and those with $E>E_c(t)$ remain in the original direction. This approach also leads to the well known definition of magnetic viscosity $S = T\rho(E_c)$ [3–5].

But due to the specific dependence of the relaxation speed Γ on the energy barrier E all terms in (5) can be evaluated analytically. Changing variables $u=\Gamma(E)=\exp(-E/T)$ in (5) and using the definition of the critical energy $E_c = T\ln\tau$, we obtain the desired expression for the time-dependent magnetization in the form

$$-\frac{\mathrm{d}m(\tau)}{\mathrm{d}\tau}=\frac{T\rho(E_c)}{\tau}\left[1+\frac{1}{\rho(E_c)}\sum_{k=1}^{\infty}\frac{\rho^{(k)}(E_c)}{k!}\times(-1)^kT^kI_k\right], \qquad (6)$$

where numerical constants I_k are given by

$$I_k=\int_0^{+\infty}(\ln x)^k\,\mathrm{e}^{-x}\,\mathrm{d}x$$

(for reference $I_0=1$, $I_1=-C$, $I_2=\pi^2/6+C$, where $C\approx 0.577\ldots$ is the Euler constant). Hence the magnetic viscosity defined in a usual way, i.e., following (3), is equal to

$$S=T\rho(E_c)\left[1+\frac{1}{\rho(E_c)}\sum_{k=1}^{\infty}\frac{\rho^{(k)}(E_c)}{k!}\times(-1)^kT^kI_k\right]. \qquad (7)$$

Eq. (6) (or (3) and (7)) represents the exact analytical expression for the time-dependent magnetization (magnetic viscosity) for any system with the known energy barrier distribution density $\rho(E)$. For practical purposes, the first two terms of the series (6) or (7) are usually sufficient due to the rapid convergence of the factorial series (if $\rho(E)$ does not change very rapidly, so that the derivatives $\rho^{(K)}(E)$ are not extremely large).

This result allows us to understand, why does the 'critical volume approach' (first term of (7)) works so well: the corrections to this approximation are usually small, because the kth term in the expansion (6) (kth correction) is proportional to $\Delta m^{(k)} \sim T^k \rho^{(k)}(E_c)/\rho(E_c)$.

For a "good" function $\rho(E)$ the relation $\rho^{(k)}(E_c)/\rho(E_c) \sim 1/\sigma_{\Delta E}^k$, where $\sigma_{\Delta E}$ is the width of the energy barrier distribution, and hence $\Delta m^{(k)} \sim (T/\sigma_{\Delta E})^k$. To estimate this quantity for room temperature ($T \approx 3 \times 10^{-14}$ erg), we consider the typical case of uniaxial particles with the anisotropy energy $E \sim KV$ (K being the uniaxial anisotropy constant and V the particle volume). For the typical value $K \approx 10^6$ erg/cm^3 even for very small particles of the size $d \approx 100$ Å and narrow size distribution with the reduced standard deviation $\sigma_d \approx 0.2$, we obtain $\sigma_{\Delta E} \approx 3 \times 10^{-13}$ erg and $T/\sigma_{\Delta E} \approx 0.1$. This relation decreases rapidly with increasing particle diameter and size distribution width.

It should also be mentioned, that (7) allows us to obtain the expression for the magnetic viscosity in the form of $\ln(\tau/\tau_0)$-series, where τ_0 is the time of the measurement's beginning. It can be done by substituting $E_c = E_{c0} + \Delta E = T\ \ln(\tau_0) + T\ \ln(\tau/\tau_0)$ and expanding the barrier density $\rho(E)$ around $E = E_0$. It would result in a more general expression in comparison with that obtained in ref. [8] using the critical volume approximation.

In conclusion, we have derived the exact analytical expression for the time-dependent magnetization for any system with the known energy barrier distribution density. It is shown, that the corrections to the critical volume approach are of order $\sim T/\sigma_{\Delta E}$ and hence are significant only for a system with narrow energy distribution density at sufficiently high temperatures.

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References

[1] R.W. Chantrell and E.P. Wohlfarth, Phys. Stat. Sol. (a) 91 (1986) 619.

[2] S.H. Charap, J. Appl. Phys. 63 (1987) 2054.

[3] P. Gaunt, Phil. Mag. 34 (1976) 775.

[4] R.W. Chantrell, M. Fearon and E.P. Wohlfarth, Phys. Stat. Sol. (a) 97 (1986) 213.

[5] R.W. Chantrell and P.R. Bissel, Nondestr. Test. Eval. 6 (1991) 1.

[6] R. Street and J.C. Wooley, Proc. Phys. Soc. A 62 (1949) 562.

[7] A. Aharoni, J. Appl. Phys. 57 (1985) 4702.

[8] M. El-Hilo, K. O'Grady and R.W. Chantrell, to be published.