Fast Switching of Magnetic Nanoparticles: Simulation of Thermal Noise Effects using the Langevin Dynamics

Dmitri V. BERKOV

***Abstract*--The most straightforward method to simulate fast switching in magnetic systems is the solution of stochastic equations of motion for magnetic moments (Langevin dynamics), where thermal fluctuations are taken into account by the thermal (random) field $\mathbf{H}^{fl}$.**

In this paper we address first an important methodical problem of this formalism: the choice of the stochastic calculus (*Ito* or *Stratonovich*). We prove that *both* Ito and Stratonovich stochastic integrals give *identical* results, despite the *multiplicative* noise present in the stochastic Landau-Lifshitz-Gilbert equation. Discussing correlation properties of $\mathbf{H}^{fl}$ (which is usually assumed to be δ-correlated both in space and time), we point out that *finite* correlation time and radius of this field can be due not only to physical reasons (heat-bath correlations), but can also arise from the finite-element representation of the continuous problem.

Afterwards we present simulation results concerning the influence of thermal fluctuations on the fast switching of magnetic nanoelements. We consider three typical situations: (i) **thermal noise influence on the switching which would happen also in the absence of thermal fluctuations (thermally assisted switching),** (ii) **thermally induced switching of the metastable states and** (iii) **changing of the switching mode as the consequence of thermal fluctuations.**

***Index Terms*— fast switching, Langevin dynamics, magnetic nanoelements, thermal noise.**

I. Introduction

Simulation of fast switching processes in magnetic systems is at present one of the hot topics in applied magnetism in general and in micromagnetics in particular. Decreasing read-write times require exhaustive understanding of corresponding remagnetization processes which happen on the nanosecond (and shorter) time scales. Rapidly increasing storage density of classical media and introduction of novel storage technologies like MRAM results in the increasing role of thermal effects which must be taken into account explicitly.

Among various methods used for studying of fast magnetization switching taking into account thermal fluctuations the Langevin dynamics is probably the most straightforward and reliable one. In this formalism the stochastic (Langevin) equations of motion for magnetic moments are solved, which allows to take into account (i) all relevant magnetic interactions (thus ensuring proper simulation of collective remagnetization modes), (ii) dynamic effects which do not influence the system energy and thus can not be included in Monte-Carlo simulations (magnetic moment precession) and (iii) thermal fluctuations using the fluctuation (random) field $\mathbf{H}^{fl}$ added to the deterministic effective field $\mathbf{H}^{det}$.

In this paper we address some general problems concerning such Langevin dynamic simulations. The paper is organized as follows. In Sec. II we discuss an important issue concerning the choice between Ito and Stratonovich interpretations of stochastic differential equations (SDE). We prove that for standard micromagnetic models *both* Ito and Stratonovich stochastic calculus lead to *identical* and physically correct results. In Sec. III discretization effects which have important consequences for dynamical simulations even without thermal noise are considered. Simulation results concerning the influence of thermal fluctuations on fast switching of magnetic nanoelements are presented in Sec. IV. Here we discuss also the choice and justification of the random field correlation properties and analyze several situation where thermal noise effects lead not only to quantitative changes, but also to qualitatively different remagnetization processes.

II. Langevin Dynamics in Micromagnetics: Equivalence of the *Ito* and *Stratonovich* Stochastic Calculus

A. A Brief Introduction to the Problem

Stochastic differential equation (SDE) which is usually solved in micromagnetics is the Landau-Lifshitz-Gilbert equation [1] for the magnetic moment motion

$$\frac{d\boldsymbol{\mu}_i}{dt} = -\gamma[\boldsymbol{\mu}_i \times (\mathbf{H}_i^{\det} + \mathbf{H}_i^{\mathrm{fl}})] - \frac{\gamma\lambda}{\mu_i}[\boldsymbol{\mu}_i \times [\boldsymbol{\mu}_i \times (\mathbf{H}_i^{\det} + \mathbf{H}_i^{\mathrm{fl}})]] \quad (1)$$

Here the precession constant γ is equal to the gyromagnetic ratio γ_0 in the small damping $\lambda << 1$. The deterministic effective field $\mathbf{H}_i^{\det}$ acting on the i-th magnetic moment $\boldsymbol{\mu}_i$ includes all the standard micromagnetic contributions (external, anisotropy, exchange and magnetodipolar interaction fields). Thermal noise is taken into account via the fluctuation (Langevin) field $\mathbf{H}_i^{\mathrm{fl}}$. In the simplest approximation its Cartesian components are assumed to be δ-correlated Gaussian random variables [1]

$$\langle H_{\xi,i}^{\mathrm{fl}} \rangle = 0, \qquad \langle H_{\xi,i}^{\mathrm{fl}}(0) \cdot H_{\psi,j}^{\mathrm{fl}}(t) \rangle = 2D \cdot \delta(t)\delta_{ij}\delta_{\xi\psi} \quad (2)$$

Manuscript received Feb. 13, 2002.

The author is with *INNOVENT* e.V. Technology, D-07745, Jena, Germany (e-mail: db@innovent-jena.de).

Digital Object Identifier 10.1109/TMAG.2002.801905

(here i, j are the moment indices and ξ, $\psi = x, y, z$). The noise power $D=(\lambda/1+\lambda^2)\cdot(kT/\gamma\mu)$ can then be evaluated from the fluctuation-dissipation theorem [2, 3].

Stochastic LLG-equation (1) does *not* (see below) automatically conserve the moment magnitude μ (in contrast to its deterministic counterpart). However, in standard micromagnetic approach μ must be kept constant, which has important consequences both for the solution of (1) (see Sec. *B*) and for the temperature region where the Langevin approach is valid: simulation temperature should be well below the Curie point T_c.

Introducing the unit moment vector $\mathbf{m} = \boldsymbol{\mu}/(M_S\cdot\Delta V)$ (M_S being the material saturation magnetization and ΔV - a discretization cell volume), the reduced magnetic field $\mathbf{h} = \mathbf{H}/M_S$, and the reduced time $\tau = t\cdot\gamma M_S$, we can rewrite (1) as

$$\dot{\mathbf{m}}_i = -[\mathbf{m}_i \times (\mathbf{h}_i^{\text{eff}} + \mathbf{h}_i^{\text{fl}})] - \lambda\cdot[\mathbf{m}_i \times [\mathbf{m}_i \times (\mathbf{h}_i^{\text{eff}} + \mathbf{h}_i^{\text{fl}})]] \quad (3)$$

Lacking the space, we can not discuss here neither the question whether this equation is a generally best choice nor how to choose the damping λ (see an interesting discussion in [4 - 6]).

The first important problem by the solution of SDE's like (1) or (3) is that they can *not* be interpreted as usual differential equations. This can be easily demonstrated using the simplest SDE describing a particle motion in a viscous medium in the presence of thermal fluctuations

$$\eta\dot{x} = F_{\text{det}}(t) + a(x,t)\cdot\xi_L(t) \quad (4)$$

Here η is the particle friction coefficient, F_{det} - the *deterministic* force. The reduced thermal force ξ_L is normally assumed to be a random Gaussian variable with correlations as in (2):

$$\langle\xi(t)\rangle = 0, \qquad \langle\xi(0)\cdot\xi(t)\rangle = 2D\cdot\delta(t) \quad (5)$$

with the fluctuation power $D \sim T$. The 'good' function $a(x,t)$ shows that the noise characteristics may be coordinate- and time-dependent.

An attempt to integrate (4) as a usual differential equation unavoidably leads to an integral of the type

$$W(t) = \int_0^t \xi(t')\cdot dt'$$

From (5) it can be easily deduced that $W(t)$ is the so called standard Wiener process [7] which is *not differentiable*. So the derivative $dW/dt = \xi(t)$ does not exist and hence in the usual sense the equation (5) including $\xi(t)$ *does not exist* also.

A rigorous method to assign a correct meaning to the relations like (4) and (1) is based on the introduction of the Wiener process differential $dW = \xi(t)dt$. Using dW, we can define integrals containing this differential

$$I = \int a(x,t)\cdot dW(t)$$

analogously to the standard Riemann-Stieltjes integrals as the limit (in the mean square sense) of corresponding partial sums

$$I = \lim_{n\to\infty}\sum_{i=1}^{n} a(x(\tau_i),\tau_i)\Delta W_i \equiv \lim_{n\to\infty}\sum_{i=1}^{n} a_i[W(t_i) - W(t_{i-1})] \quad (6)$$

The points τ_i where the values of the integrand are evaluated lie, as usual, somewhere inside the interval $[t_{i-1}, t_i]$. The problem is that not only the values of partial sums, but the limit (6) itself *depends on the choice of intermediate points* τ_i (see [7], Chap. 3 for a simple but impressive example). This fact is in a heavy contrast to the standard analysis where the *independence* of the limit of Darbu sums on the intermediate point choice is one of the cornerstones by building the integral calculus**.**

The only way to cope with this problem is to introduce standard choices for intermediate points and to find the best one from the statistical physics point of view. Currently there exist two such choices: (i) intermediate points $\tau_i = t_{i-1}$ coinciding with the *beginning* of the intervals lead to the *Ito* stochastic integral and (ii) $\tau_i = (t_{i-1} + t_i)/2$ lying in the *middle* of the intervals result in the *Stratonovich* stochastic calculus.

It is well known that Ito and Stratonovich interpretations of SDE's lead to *different* solutions if the noise is *multiplicative* - i.e., the random term is *multiplied* by some function of the system variables. In this case usually the *Stratonovich* interpretation provides physically correct results, recovering, e.g., important properties of physical random processes obtained with more general methods [7].

B. Equivalence of the Ito *and* Stratonovich *Stochastic Calculus for Models with* ***Constant*** *Moment Magnitudes*

The noise in the Langevin equation (1) is *multiplicative* - due to the vector products the projections of the random field $\mathbf{H}^{\text{fl}}$ are *multiplied* by the magnetic moment projections. This fact was noticed already in the pioneering paper of Brown [1] who suggested to use the *Stratonovich* interpretation of (1).

The question was raised again in the last decade when numerical simulations using (1) became available and many research groups started to perform Langevin dynamics studies of remagnetization processes in various systems (see [2, 8-12], etc). For such simulations the choice of the stochastic calculus is of primary importance, because different numerical methods converge to different stochastic integrals: The Euler scheme and simple implicit methods converge to the Ito solution, Heun and Milstein schemes are known to converge to the Stratonovich limit [13] and Runge-Kutta (RK) schemes can converge to anything (including the in-between cases) depending on their coefficients [14]. Most authors [2, 9, 10] and commercial micromagnetic packages [15, 16] use Heun and RK methods converging to the Stratonovich solution, but several groups employ the Ito-converging Euler [8, 11] and implicit schemes [12]. The last two papers were seriously criticized in [2] where it has been claimed once again that only the Stratonovich interpretation ensures the physically correct solution of (3).

In this subsection we shall prove that for the SDE's (1)-(3) there is *no difference* between the Ito and Stratonovich interpretations *if the magnitude of magnetic moments is assumed to be constant.* This is the case in the overwhelming majority of magnetic models including (but not limited to) classical Heisenberg and RKKY models, in models of single-domain magnetic particle systems [17, 18] and in standard micromagnetics [19] (which is most relevant for our purposes).

First we note that the fluctuation field in the dissipation term on the right-hand side of (3) can be omitted; although the particular realizations of system trajectories will be different then, the *average* system properties (the only being of practical interest) remain the same if the noise power D is rescaled correspondingly [2, 3]. Thus we can restrict ourselves to the study of a simpler stochastic equation

$$\dot{\mathbf{m}}_i = -[\mathbf{m}_i \times (\mathbf{h}_i^{\text{eff}} + \mathbf{h}_i^{\text{fl}})] \quad (7)$$

Next we make use of the well known fact [20], that if one adds to the system of stochastic ODE's

$$\dot{x}_i = A_i(\mathbf{x}, t) + \sum_k B_{ik}\xi_k \quad (8)$$

the *deterministic* drift term $\sum_{jk} B_{jk}(\partial B_{ik}/\partial x_j)$, then the *Ito* solution of this modified system is equivalent to the *Stratonovich* solution of the initial system (8). Comparing (8) with the LLG-system (7), we can see that in our case the matrix **B** is $B_{ik} = D\sum_j \varepsilon_{ijk} m_j$, so that the above mentioned drift reduces to $d\mathbf{m}_i/d\tau = -2D\mathbf{m}_i$.

This drift is directed *along* the magnetic moment $\mathbf{m}_i$ thus trying to change its *magnitude* which is forbidden by the model. For this reason this term *must* be discarded which means that *for stochastic dynamics of models with rigid dipoles (with constant magnitudes) there is no difference between the Ito and Stratonovich solutions of such stochastic ODE's*.

The mathematical reason for this equivalence is that *Cartesian* coordinates of magnetic moments are *not* independent due to the condition $|\mathbf{m}| = 1$. Independent variables are *spherical* coordinates (θ,ϕ) of **m**. After transition to these coordinates the *stochastic* part of (7) which we have to analyze reads [1,3]

$$\dot{\theta} = h_\varphi^{\text{fl}}, \qquad \dot{\varphi} = -h_\theta^{\text{fl}}/\sin\theta$$

(we have omitted the moment index *i* for simplicity), so that the matrix **B** responsible for the drift mentioned above is

$$B = \begin{pmatrix} B_{\theta\theta} & B_{\theta\varphi} \\ B_{\theta\varphi} & B_{\varphi\varphi} \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -1/\sin\theta & 0 \end{pmatrix}$$

It is straightforward to verify that the corresponding drift is exactly zero: $\sum_{jk} B_{jk}(\partial B_{ik}/\partial x_j) = 0$ (here $i,j,k = 1,2$ and $x_1 = \theta$, $x_2 = \phi$). Hence we arrive at the same result that Stratonovich and Ito stochastic integrals are equivalent in this case. Numerical examples supporting this conclusion can be found in [21].

There exist an opposite statement made in [2] which is based on the usage of the Fokker-Planck equation (FPE) for the evolution of the probability distribution of the magnetization orientation $P(\mathbf{m},t)$. In FPE derived from the *Ito* version of the Langevin equation an additional drift term $\partial(\mathbf{m}P)/\partial\mathbf{m}$ arises when compared with the *Stratonovich* case (see p.14940 in [2]). However, because Cartesian coordinates of **m** are *not* independent, one can *not* use the FPE written in these coordinates to compare Ito and Stratonovich without introducing the restriction $|\mathbf{m}| = 1$ explicitly. In particular, the additional drift term $\partial(\mathbf{m}P)/\partial\mathbf{m}$ in the Ito interpretation of FPE should be *omitted* because it leads to the drift of the probability density *along* the magnetization vector. This can be clearly seen after transition to *spherical* coordinates (m, θ, ϕ) of **m** where this drift term reduces to $\partial(mP)/\partial m$, thus trying to change the moment magnitude, which is *forbidden* by the model.

We note in passing that by numerical solution of (7) *Cartesian* coordinates are often preferred, because no instabilities present in spherical coordinates near the polar axis can occur. During such simulations one has to normalize the moment vector $\mathbf{m}_i$ after each integration step (and also by evaluating the derivatives at the intermediate points) in order to conserve the moment magnitude thus restoring the equivalence of the two stochastic integrals discussed above.

III. Discretization Effects in Dynamic Micromagnetic Simulations

In this section we turn our attention to the influence of discretization effects (finite-element representation of the continuous problem) on the ***dynamic*** switching behaviour of magnetic nanoelements in numerical simulations.

To study these effects we have chosen a relatively simple and well defined problem. We consider a dynamic switching of a thin nanoelement with lateral sizes L_x x L_z = 400 x 600 nm and thickness h = 5 nm. The 0*xz*-plane of our coordinate system lies in the element plane and the 0*x*-axis is directed horizontally relative to images shown in the figures. To avoid uncontrollable effects due to the random grain anisotropy and the polycrystalline sample structure discretization, we have set this anisotropy to zero leaving only a shape anisotropy. Other magnetic parameters of the element are: saturation magnetization M_S = 1000 G and exchange stiffness constant $A = 10^{-6}$ erg/cm.

We have simulated the switching of such an element integrating the LLG equation (3) using an optimized Bulirsch-Stoer algorithm with the adaptive step-size control. We start from the *S*-type remanent state (see first gray-scale image in Fig. 1) applying at t = 0 the external field $\mathbf{H} = H_z\mathbf{e}_z$ with H_z = – 200 Oe; this field is well beyond the corresponding quasistatic switching field which for this problem was calculated to be $H_{sw} \approx$ – 80 Oe. To study the discretization effects *only* we have excluded the thermal noise influence by setting the temperature T to zero. Simulations were done for five sequentially refined grids with the same (1:1) aspect ratio of the grid cells: N_x x N_z = 40 x 60, 60 x 90, 80 x 120, 120 x 180, 200 x 300.

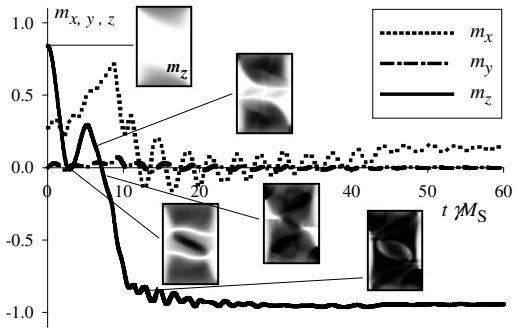

Fig. 1 Switching of a thin rectangular 'soft' magnetic element (400 x 600 x 5 nm) with a *low* damping (λ = 0.01) in a negative field H_z = –200 Oe starting from the *S*-type remanent state

First we discuss results for the most interesting low damping case. The switching progress for to λ = 0.01 is shown in Fig. 1 as time dependencies of all magnetization components and m_z-gray-scale maps for several time moments (discretization N_x x N_z = 120 x 180). The switching starts with the reversal of closure domains near the short element borders. The subsequent reversal of the central domain is accompanied by the strong oscillation effects due to a low dissipation so that its magnetization direction changes several times during the relaxation.

The reversal of narrow domains near the long sides completes the remagnetization.

To characterize the importance of the discretization effects we have compared results for several grids listed above. In Fig. 2 we present corresponding $m_x(t)$-dependencies, because in our geometry the influence of the discretization effects can be most clearly seen on this projection.

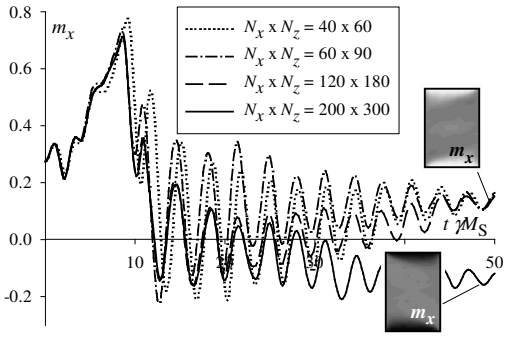

Fig. 2 m_x time dependencies for the switching process shown in Fig. 1 simulated using various grids

The main conclusion which can be immediately drawn from the analysis of Fig. 2 is that the remagnetization curves do not converge to any limiting curve up to the finest grid simulated (N_x x N_z = 200 x 300). The effect is even qualitative, as it can be seen from the comparison of final states (m_x-gray-scale maps on the right in Fig. 2) for all discretizations N_x x $N_z \leq$ 120 x 180 and for N_x x N_z = 200 x 300.

This discrepancy can not be attributed to an insufficient discretization of the interaction (energy) terms. First of all, already for a moderate grid 80 x 120 the cell sizes $\Delta_x = \Delta_z$ = 5 nm are two times smaller than our characteristic micromagnetic length $l_{dem} = (A/M_S^2)^{1/2}$ = 10 nm, so that further grid refinement should lead to a fast convergence of results. Second, we have verified which discretization is sufficient for the solution of quasistatic problems by simulating corresponding hysteresis loops: starting from the grid N_x x N_z = 60 x 90 such loops did not change (in frames of the computer accuracy) when the grid was refined further.

The reason for a significant modification of the switching process by the grid refinement is a strong influence of magnetic excitations with a short wavelength. The non-linear (due to the double vector product) dissipation term in the LLG equation leads to the generation of excitations with shorter wavelengths when the long-wave magnons decay. These short-wave excitations produce magnons with still smaller wavelength etc. A given grid is obviously unable to support excitations with a wavelength smaller than its cell size. Hence such a can not provide adequate picture of the switching process starting from the time moment when such short-wave excitations start to play an important role.

To support this conclusion, we have plotted in Fig. 3 the differences between $m_x(t)$-dependencies for two subsequent grid refinements. It can be seen, that for finer grids these differences start to diverge later (proving that we are dealing with a purely dynamical effect). This means that on a given lattice dynamical simulations are valid up to a certain *maximal* time, after which the effect of short-wave fluctuations not supported by this lattice becomes important. For maximal allowed difference δ_{max} = 0.05 between the average magnetization projections the arrows under the time axis in Fig. 3 show the maximal allowed simulation duration for the given discretizations.

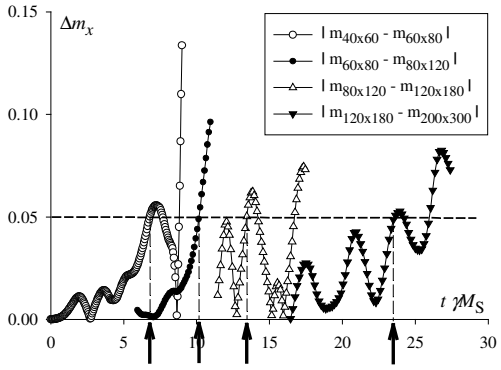

Fig. 3. Differences between $m_x(t)$-dependencies for two subsequent grid refinements as indicated in the legend. Arrows indicate the *maximal* allowed simulation time for corresponding grids when the maximal allowed error threshold is set to δ = 0.05.

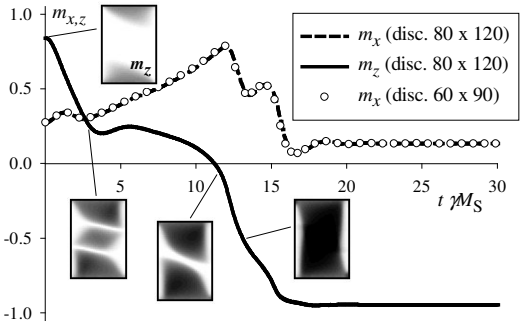

Fig. 4 Switching of the same element as shown in Fig. 1, but for with a *moderate* damping (λ = 0.1). No differences between the switching processes for progressive grid refinements could be observed

For systems with larger dissipation the discretization effect demonstrated above may be absent due to a much smaller decay times of the short-wave magnons. For this reason they may not play any significant role by the switching process. This is demonstrated in Fig. 4 where results of the same simulations as above, but for λ = 0.1 are presented. No noticeable changes in the behaviour of any magnetization projection could be detected starting from the discretization N_x x N_z = 60 x 90.

IV. Thermal Noise Influence on the Switching Dynamics of Nanolelements

A. Correlation Properties of a Random Noise

Another very important methodical question is the establishment of correlation properties of the random field $\mathbf{H}^{fl}$. Up to our knowledge, virtually all papers dealing with stochastic micromagnetic simulations use the simplest approximation given by (2). However, one should keep in mind that this formulae were derived, strictly speaking, for thermal fluctuations of a magnetic moment of a single and absolutely single-domain particle [1] for time intervals much larger than characteristic times of heat bath correlations.

For this reason it is clear that for stochastic simulations of micromagnetic problems, where one deals with complicate systems with several interaction types and on very different time scales, correlation properties (2) should be revised. The first obvious situation, when such a revision is necessary, is the switching on the time scale compared with the heat bath correlation times which are usually estimated to lie in the picosecond region [1-3]. However, such ultrafast switching processes are, at least at present, not the mainstream research subject, so we shall not discuss this case further.

Another by far more important reason for non-trivial correlations present in the random field components is the discretization of the continuous problem itself. Namely, as it was mentioned in Sec. III, the given grid is unable to support excitations with the wavelength smaller than the grid cell size. From the point of view of thermal fluctuations this means that corresponding thermal magnons can not be included into simulations explicitly. Such short-wave magnons may still have the mean free path much *larger* than the cell size, thus leading to substantial time and space correlations of the random field acting on magnetic moments of the simulation lattice. This circumstance would first modify the δ-functional time and space correlation properties (2) of the random noise. The second effect would be the modification of the random noise *power* given by the coefficient D in (2), because the latter should include all the internal degrees of freedom which the lattice cell magnetization might possess. Neglecting these internal excitations leads to the underestimation of the noise power and to its incorrect dependence on the cell size (as ΔV^{-1}).

At present we are not able to give a correct quantitative description of the effects mentioned above, so we shall use in the simulations presented below the simplest ansatz (2). However, one should clearly understand that after retrieving correct correlation properties of the random noise we expect at least substantial quantitative corrections to the switching behaviour obtained within this simplest approximation.

B. Thermally Assisted Switching

If the switching would take place without thermal noise also (like the process shown in Fig. 1) then thermal fluctuations mostly do not cause any qualitative changes in the switching process. They lead to (i) its acceleration due to the more efficient dissipation of the system energy and - for the same reason - (ii) to additional damping of magnetization oscillations. In this case we speak about a thermally *assisted* switching

C. Thermally Induced Switching

If in the absence of thermal noise a magnetization state is metastable and hence would not decay by itself, its switching may be caused by thermal fluctuations. A typical example of such a process is shown in Fig. 5 and 6 (note the break on the t-axis !), where we have used the magnetic element with the same geometry and magnetic parameters as in previous examples. Starting from the S-type remanent state, we have calculated the *metastable* static equilibrium state in the negative field $H_z = -77$ Oe, which magnitude is slightly *smaller* than the switching field magnitude for this case ($H_z^{sw} = 79.5 \pm 0.5$ Oe).

Afterwards we have switched the temperature on, keeping the external field constant. After the initial 'heating' (which can be seen in Fig. 6 as the fast increase of all energy contributions) the system reaches thermal equilibrium inside the energy minimum corresponding to this metastable state and remains in this minimum till $\tau \approx 80$. Then a thermally induced transition to a more stable state with $\langle m_z \rangle < 0$ occurs. Corresponding time moment obviously depends on the noise realization, but the *average* transition time is uniquely determined by the initial magnetization configuration and the external field.

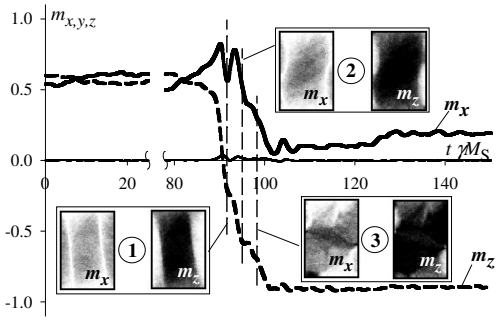

Fig. 5. Thermally caused switching of a nanoelement with the same geometric and magnetic parameters as in previous examples in the external field $H_z = -77$ Oe (which magnitude is smaller than the switching field)

It is interesting to note that this thermally induced transition occurs via three well defined states shown as 1, 2 and 3 in Fig. 5 and marked with vertical dashed lines in Fig. 6, where the energy time dependence during the transition is shown. The first and third states provide local maxima to the exchange energy (due to two sharp vertical 180° domain walls in state 1 and a long zigzag ≈ 90° wall in state 3), being at the same time local minima for the stray field energy. The 2nd state, in contrast to them, minimizes the exchange energy (almost no domain walls), but has a maximal demagnetizing energy because magnetization along the vertical element sides is oriented nearly perpendicular to them, creating large 'magnetic charges'. This sequence of intermediate states was observed for all realizations of the transition studied here, irrespective of the transition time and the external field value.

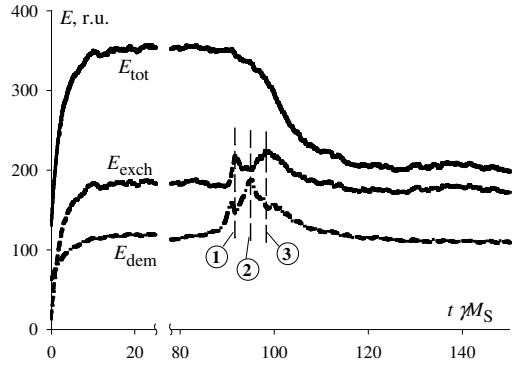

Fig. 6 Energy dependence during the magnetization relaxation shown in Fig. 5. States marked with vertical dashed lines correspond to those which magnetization configuration is shown in Fig. 5 using gray-scale maps.

D. Thermal Noise Influence on the Switching Mode

Another important effect of a thermal noise which may qualitatively influence a switching process is a change of the switching *mode.*

A simple situation when it takes place is demonstrated in Fig. 7 and 8. If we simulate at $T = 0$ the reversal of the same nanoelement as considered above, but starting from the flower remanent state (Fig. 7), we will find that the in this case the negative field $H_z = -200$ Oe is unable to switch the element completely due to the very unfavorable switching mode. Namely, the four growing edge domains form nearly 360°-domain walls along the two rectangle symmetry axes, so that a much higher field would be necessary to destroy this walls and switch the element completely.

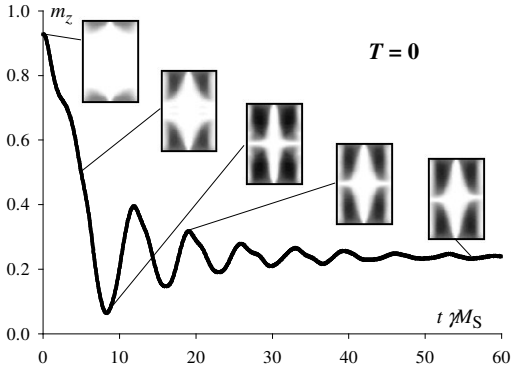

Fig. 7 Switching of the same element as shown in Fig. 1, but starting from the *flower* remanent state at $T = 0$. Note that the negative field H_z = -200 Oe is insufficient to switch the element completely

When simulating the switching process starting from the same flower state, but at a *finite* temperature (see Fig. 8, where $T = 300$), a completely different process takes place. During the thermal equilibration of the system ($\tau < 20$) the relatively unstable symmetric flower state is converted to the *S*-type remanent state (spontaneous symmetry breaking due to thermal fluctuations). When the negative field $H_z = -200$ Oe is then switching on at $\tau = 20$, a fast switching of this state occurs in the same way as shown in Fig. 8.

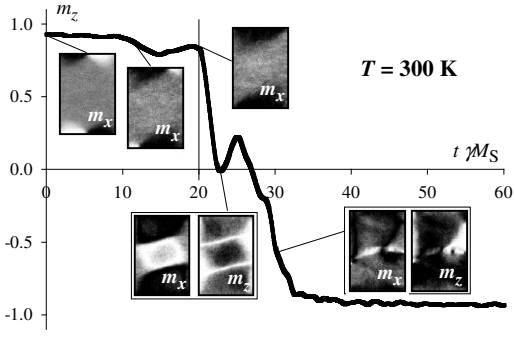

Fig. 8 The switching process for the same element as shown in Fig. Sw_T0, but for the temperature T = 300 K. The negative field H_z = -200 was switching on after the equilibrium state has been reached. See text for detail.

V. Conclusion

From the examples presented above it should be clear that numerical simulations using the Langevin dynamics formalism are a powerful tool for investigation of fast magnetic switching. However, it should be also clear that obtaining quantitatively correct results – actually the main goal of such simulations – is impossible without proper understanding of all the discretization effects, which are important for simulations both with and without thermal fluctuations.

Acknowledgment

The author would like to thank Dr. N.L.Gorn for programming assistance, Prof. J. Miltat and Dr. S. Demokritov for many useful discussion and Prof. P. Görnert for his interest and encouraging support.

References

[1] W.F. Brown, Jr., „Thermal fluctuations of a single domain particle“, *Phys. Rev*., vol. 130, pp. 1677-1686, 1963

[2] J.L. Garcia-Palacios, F.J. Lazaro, "Langevin-dynamics study of the dynamical properties of small magnetic particles", *Phys. Rev*., vol. B58, pp. 14937-14958, 1998

[3] H.-B. Braun, "Stochastic magnetization dynamics in magnetic nanostructures: from Neel-Brown to soliton-antisoliton creation", in: *Structure and Dynamics of Heterogeneous Systems*, Eds. P. Entel & D. Wolf, World Scientific, 2000

[4] H. Suhl, "Theory of the magnetic damping constant", *IEEE Trans. Magn*., vol. MAG-34, pp. 1834-1838, 1998

[5] X. Feng, P.B. Fischer, "Coarse-graining Landau-Lifshitz damping", *J. Appl. Phys*., vol. 80, pp. 6988-6990, 2001

[6] A. Thiaville, J. Miltat, J. Ben Youssef, "Dynamics of vertical Bloch lines in bubble garnets: experiments and theory", *Europ. Phys. J. B*, vol. 23, pp. 37-47, 2001

[7] S. Gardiner, *Handbook on Stochastic Processes*, Springer-Verlag, Berlin, 1997

[8] K. Zhang, D.R. Fredkin, "Thermal relaxation of clusters of interacting Stoner-Wohlfarth particles", *J. Appl. Phys*., vol. 87, pp. 4795-4797, 2000

[9] D.V. Berkov, N.L. Gorn, P. Görnert, "The Langevin-dynamics simulation of interacting fine magnetic particle system", *J. Magn. Magn. Mat.*, vol. 226, pp. 1936-1938, 2002

[10] W. Scholz, T. Schrefl, J. Fidler, "Micromagnetic simulations of thermally activated switching in fine particles", *J. Magn. Magn. Mat.*, vol. 233, pp. 296-304, 2001

[11] A. Lyberatos, R.W. Chantrell, "Thermal fluctuations in a pair of magnetostatically coupled particles", *J.Appl.Phys*., vol. 73, pp. 6501-6503, 1993

[12] Y. Nakatani, Y. Uesaka, N. Hayashi, H. Fukushima, "Computer simulation of thermal fluctuation of fine particle magnetization based on Langevin equation", J. Magn. Magn. Mat., vol. 168, pp. 347-352, 1997

[13] E.J. McShane, *Stochastic calculus and stochastic models*, Acad. Press, New York, 1974

[14] W. Rümelin, "Numerical treatment of stochastic differential equations" *SIAM J. Numer. Anal*., vol. 19, pp. 604-613, 1982

[15] *Advanced Recording Model*, package for micromagnetic simulations, Euxine technologies, http://www.euxine.com

[16] M. Scheinfein, Landau-Lifshitz-Gilbert Micromagnetics Simulator, http://www.dancris.com/~llg

[17] K. Binder, A.P. Young, „Spin glasses: experimental facts, theoretical concepts, and open questions“, *Rev. Mod. Phys*., vol. 58, pp. 801-976, 1986

[18] M.F. Hansen, S. Morup, "Models for the dynamics of interacting magnetic nanoparticles", *J. Magn. Magn. Mat.*, vol. 184, pp. 262-274, 1998

[19] W.F. Brown, *Micromagnetics*, N.Y., Wiley, 1963

[20] P.E. Kloeden, E. Platen, *Numerical Solution of Stochastic Differential Equations,* Springer-Verlag, Berlin, 1995

[21] D.V. Berkov, N.L. Gorn, P. Görnert, "Magnetization dynamics in nanoparticle systems: numerical simulation using Langevin dynamics", to appear in: *phys. stat. sol. (b)*