

# Micromagnetic studies of quasistatic and dynamic properties of densely packed hexagonal nanodisk arrays

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## Abstract

Densely packed arrays of magnetic platelets with sizes in the sub- $\mu\text{m}$  region are promising candidates for magnetic metamaterials due to the possibility to control their dynamic properties both via their geometry (shape anisotropy) and magnetization configuration (which can be changed by the applied field). Simulations of these dynamical properties are extraordinary important for the prediction of the metamaterials features prior to its preparation. Here we present an example of the micromagnetic characterization of such a system for the case of a hexagonal array of thin nanodisks. Comparing our simulation results with experimental data obtained by various measurements techniques, we show that numerical simulations represent a reliable tool for the prediction of both static (hysteresis loops) and dynamic (excitation spectra) properties of magnetic metamaterials.

## 1. Introduction

Large-scale periodic arrays of thin magnetic nanoelements (disks, rectangles, triangles etc.) with lateral sizes  $\sim 10 - 10^3$  nm and thickness  $\sim 10$  nm attract increasing attention during the last years due to the possibility to change their dynamical properties applying an external field *after* the array preparation. This field can affect the magnetization state of array elements, thus qualitatively changing the spectrum of magnons (spin waves) of such an array. This feature is very attractive for the development of magnonic devices with adjustable properties, like tunable filters and resonators in the GHz frequency region, logic gates based on the domain wall dynamics etc. Taking into account that preparation and experimental characterization of such an array is a challenging and laborious task, it is highly desirable to have a possibility to predict its properties *before* it has been actually produced. A powerful tool for such a prediction are micromagnetic simulations of both hysteresis loops (change of the array magnetization in the slowly varying field) and the dynamic response of the of nanoelement lattices.

Micromagnetic formalism [1] allows to evaluate the total magnetic energy  $E_{\text{tot}}$  of a ferromagnet (FM) if its geometry, material parameters and magnetization state are known. This total energy consists of the energy  $E_{\text{ext}}$  in the external field  $\mathbf{H}_{\text{ext}}$ , magnetocrystalline anisotropy energy  $E_{\text{an}}$ , exchange stiffness energy  $E_{\text{exch}}$  and the magnetodipolar interaction energy  $E_{\text{dip}}$ . All these energies depend on the FM magnetization state  $\{\mathbf{M}(\mathbf{r})\}$ , so that minimizing the energy functional with respect to  $\{\mathbf{M}(\mathbf{r})\}$ , an *equilibrium* magnetization state can be found. For all practically relevant cases such a minimization requires the system discretization and subsequent application of numerical minimization methods [2].

*Dynamical* system response can be calculated using the equation of motion of the system magnetization, which in the most widely used Landau-Lifshitz-Gilbert form reads (below  $\gamma$  denotes the gyromagnetic ratio,  $M_S$  - the FM saturation magnetization and  $\lambda$  - the damping constant):

$$\frac{d\mathbf{M}}{dt} = -\gamma \cdot [\mathbf{M} \times \mathbf{H}^{\text{eff}}] - \lambda \cdot \frac{\gamma}{M_S} \cdot [\mathbf{M} \times [\mathbf{M} \times \mathbf{H}^{\text{eff}}]] \quad (1)$$

The effective field can be evaluated as the negative functional derivative of the system energy over the magnetization:  $\mathbf{H}_{\text{eff}} = -(1/V) \cdot \partial E / \partial \mathbf{M}(\mathbf{r})$ . Integration of this differential equation (DE) (which after

the system discretization is converted into a system of DEs describing the magnetization motion for each discretization cell) in all practically interesting cases should be also performed numerically [2].

## 2. Hysteresis loops for a hexagonal array of nanodisks

All simulations reported here have been performed using the MicroMagus package [3]. In order to be able to compare our results with experiment [4] we have simulated the hexagonal array of nanodisks with the disk diameter  $D = 370$  nm, distance between the disk centers  $L = 390$  nm, disk thickness  $h = 20$  nm. Magnetic material parameters ( $\text{Ni}_{87}\text{Fe}_{13}$  alloy with the negligibly small grain anisotropy) – magnetization  $M_S = 715$  Oe and exchange constant  $A = 1 \times 10^{-6}$  erg/cm have been determined using the saturation magnetization of the sample and the spin wave frequencies measured on the continuous film used for the array preparation. Hysteresis loop measured by the magneto-optical Kerr effect (MOKE) has a shape typical for the vortex formation in a nanodisk (open circles in Fig.1): the vortex is formed in the disk center for  $H_{\text{ext}} \approx 0$  (sharp magnetization decrease from  $m_H \approx 1$  to  $m_H \approx 0$ ), and moves gradually towards the disk edge when the field is decreased further (gradual magnetization decrease), until it is expelled out of the disk at some negative field (second sharp decrease to  $m_H \approx -1$ ).

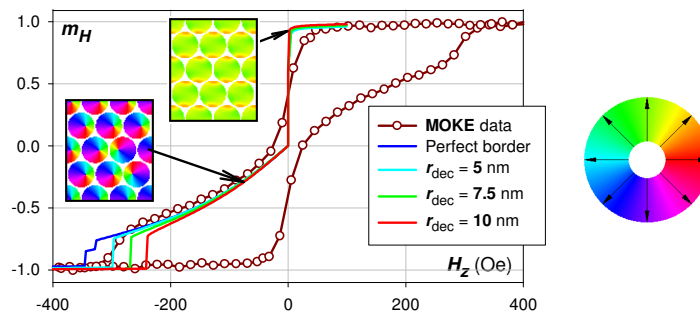


Fig. 1: Simulated and measured (MOKE) hysteresis loops for a hexagonal nanodisks array with parameters given in the text. For simulated loops only the upper hysteresis branch is shown. For negative fields simulations were started from the vortex state. Insets show magnetization maps for a typical quasi-uniform magnetization state and a vortex state (color coding of the magnetization directions is presented on the color wheel on the right).

Rigorous simulations of the transition ‘positive’ uniform state  $\rightarrow$  vortex are hardly possible: it corresponds to a spontaneous symmetry breaking of the uniform state and hence strongly depends on the details of the nanodisk edge roughness, which are largely unknown. Contrary, the transition vortex  $\rightarrow$  ‘negative’ uniform state is fully deterministic, so that its reproduction by simulations means that system parameters have been chosen properly. In our case, where geometry and material parameters are known (see above), the only adjustable parameter is the magnetization decrease near the disk edges due to the array patterning. To model this decrease, we have chosen the exponential profile  $m(r) = 1 - \exp\{(r-R_c)/r_d\}$ , where  $r$  is the distance to the disk center,  $R_c$  - disk radius,  $r_d$  is the width of the area near the nanodisk edges, where  $m$  is substantially decreased. The vortex expulsion is governed by the demagnetizing field, which strongly depends on the magnetization profile near the edges. Hence we expect that the simulated expulsion field substantially depends on  $r_d$ . Simulated loops (Fig.1) confirm this assumption, and a very satisfactory agreement is achieved for a reasonably small value  $r_d = 5$  nm. We also point out that the hysteresis slope for  $H_z < 0$  is also nearly perfectly reproduced by simulations, so that the static remagnetization of our system can be considered as being fully understood.

## 3. Dynamical response of a hexagonal array of nanodisks to a homogeneous *ac*-field

Array response to a weak oscillating field (in presence of a constant field  $\mathbf{H}_0$ ) is modelled using the Eq. (1) with the field pulse method. In this method, we first find the equilibrium magnetization state in the field  $\mathbf{H}_0$ . Then we ‘apply’ to the system in this state a trapezoidal field pulse with sufficiently small rise and fall times, so that the pulse Fourier spectrum contains significant power up to the highest frequency, for which the relevant system eigenmodes are expected. We simulate the system response to

this pulse, setting  $\lambda = 0$ , so that the magnetization oscillations continue indefinitely after the pulse is switched off. This method allows a precise determination of the resonant mode frequencies which should be observed in the homogeneous  $ac$ -field by the ferromagnetic resonance (FMR) method.

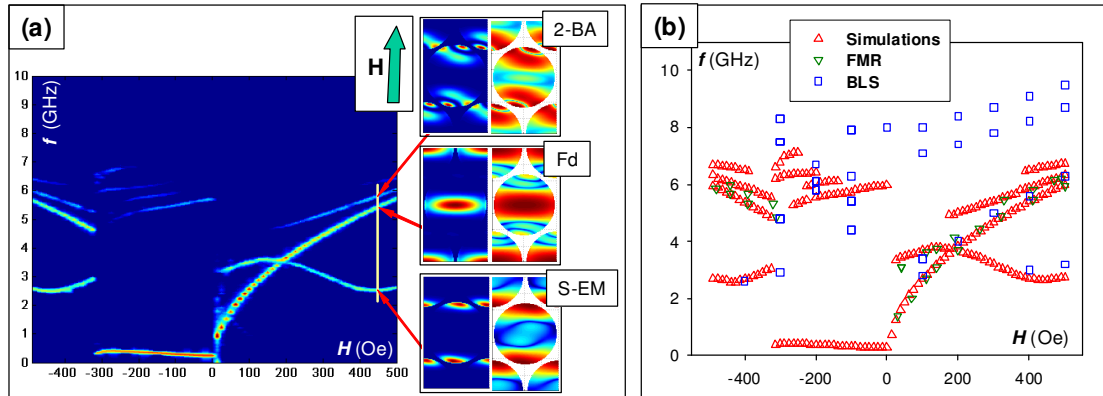


Fig. 2: (a) Simulated FMR spectra for the array of nanodisks discussed above ( $\mathbf{H}_0$  oriented perpendicular to the disk rows). Color maps on the right show spatial distributions of the oscillation power for each mode on linear and log scales. (b) Comparison of simulated frequencies (red triangles up) to experimental data (FMR frequencies - green triangles down and BLS frequencies - blue squares).

Simulation results for  $\mathbf{H}_0$  directed perpendicular to the nanodisk rows is shown in Fig. 2a, where the color plot of the power spectra vs  $H_0$  is displayed together with spatial maps of the oscillation power for each mode. Several interesting dynamical features are observed. In particular, for this field direction a clearly pronounced symmetric edge mode exists, which non-monotonic frequency dependence on  $H_0$  is due to the interplay between  $\mathbf{H}_0$  and self-demagnetizing field  $\mathbf{H}_{\text{dem}}$  of the nanodisk (see, e.g., [5]).  $\mathbf{H}_{\text{dem}}$  also leads to the highly inhomogeneous power distribution for the fundamental mode.

Comparison to the experimental data (Fig. 2b) reveals, that for the quasi-uniform magnetization state (e.g., for  $H_0 > 0$ ) frequencies of three FMR bands observed experimentally (green triangles down) agree with simulations very well. Simulations also reproduce the lower frequency branches measured by the Brillouin light scattering (BLS) technique, where the thermally excited spin waves are studied (blue open squares). Upper BLS branches are not seen in simulations, what most probably means that these modes can not be excited by a homogeneous external field.

#### 4. Conclusion

In this paper, using an array of densely packed magnetic nanodisks as an example, we have shown that micromagnetic simulations are a reliable tool for predicting quasistatic and dynamic magnetic properties of patterned nanoelement arrays, thus being a valuable tool for studying magnetic metamaterials.

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